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Searching for periodic solutions with central symmetry in Hill problem*

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We consider Hill problem (see [1, 2]) describing the motion of massless body near the minor of two active masses as a regular perturbation of Kepler problem in uniformly rotating (sinodical) frame. It makes possible to apply the classical method of Hamiltonian normal form [3] for searching generating solutions of families of periodic orbits. The essential difference of the Hill problem from the well-known Restricted Three Body Problem (RTBP) is that canonical equations of motions are invariant under the group of linear transformations of the extended phase space with generators:

$$\Sigma_1 : (t, x_1, x_2, y_1, y_2) \to (-t, x_1, -x_2, -y_1, y_2)\,,$$
$$\Sigma_2 : (t, x_1, x_2, y_1, y_2) \to (-t, -x_1, x_2, y_1, -y_2)\,,$$

This fact allows two state that the set of periodic solutions can be divided into following subsets:

- asymmetric orbits, which change under any transformation;
- singly symmetric orbits, which are invariant under transformation Σ_1 or Σ_2;
- centrally symmetric orbits, which are invariant under composition $\Sigma_{12} \equiv \Sigma_1 \circ \Sigma_2$ only;
- doubly symmetric orbits, which are invariant under any transformation.

Earlier [2, 4], periodic solutions with any type of symmetry but central were computed. This work is an attempt to find generating solutions with central symmetry and to continue them into periodic solutions of the Hill problem.

Just now the following steps are realized:

1. generalized Hill problem Hamiltonian with small parameter ε is rewritten in Delaunay variables;
2. the procedure of invariant Hamiltonian normalization up to the second order is applied;
3. the condition on existing of generating solutions can be written in the form

$$F(e, p, q) \sin k\varpi = 0,$$

   where e is an eccentricity, ϖ is an argument of the pericenter, $p + q \in \{1, 2, 4\}$. Function $F(e, p, q)$ is smooth for $e \in [0; 1)$. The parameter k equals to 2 for $p + q = 1$ or 2, equals to 4 for $p + q = 4$.

The specific of the generalized Hill problem leads to the evidence that it is possible to successfully continue such generating solutions into the Hill problem periodic orbits, which major semi-axis a is less then 1, or $p, q \in \mathbb{N}$. For centrally symmetric generating solutions it is possible if $p > 2$, p is odd, and $\varpi \neq k\pi/4$, $k \in \mathbb{N}$. A suitable for continuation generating solution corresponds to the only values $p = 3$ and $e \approx 0.8525432355$.

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