Applications of Computer Algebra – ACA2018
Santiago de Compostela, June 18–22, 2018

Schutzenberger transformation on the three-dimensional Young graph

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The Schutzenberger transformation on Young tableaux, also known as "jeu de taquin", was introduced in [1]. This transformation allows to solve different problems of enumerative combinatorics and representation theory of symmetric groups. Particularly, it can be used to calculate the Littlewood-Richardson coefficients [2].

It is known [3] that a limit distribution of Plancherel probabilities on the front of large Young diagrams of size $n, n \to \infty$ has the following density function known as semicircle distribution:

$$d\mu(u) = \frac{\sqrt{4-u^2}}{2 \cdot \pi},$$

where u is one of Vershik-Kerov coordinates: $u = \frac{x-y}{\sqrt{n}}$. Later it was proved [4] that the coordinates of Schutzenberger path ends are distributed according to the semicircle distribution as well.

However, there are no known limit distribution function of the coordinates of three-dimensional Schutzenberger path ends. Moreover, there are no known 3D analogues of the central Plancherel process and RSK correspondence. In this work we made an attempt to fill this gap by conducting some numerical experiments on the three-dimensional Young graph.

Also we considered a special randomized variant of the Schutzenberger transformation. It was found that this approach can be used to calculate the co-transition probabilities on the Young graph, which in turn gives a possibility to calculate the ratios of dimensions of 3D Young diagrams. Note that the exact dimensions of 3D Young diagrams cannot be calculated directly.

Keywords: Young tableaux, Young diagrams, Schutzenberger transformation, Jeu de taquin, Limit shape

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