

## Singular Initial Value Problems for Quasi-Linear Ordinary Differential Equations

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We discuss existence, non-uniqueness and regularity of solutions of initial value problems for quasi-linear ordinary differential equations where the initial condition corresponds to an impasse point [4] of the equation. With a differential geometric approach [1, 3], we reduce the problem to questions in dynamical systems theory. As an application, we discuss in detail second-order equations of the form  $g(x)u'' = f(x, u, u')$  with an initial condition imposed at a simple zero of  $g$ . This generalises results by Liang [2] and also makes them more transparent via our geometric approach.

**Keywords:** Quasi-linear ordinary differential equations, geometric theory, initial value problem, existence and (non-)uniqueness of solutions, regularity

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### References

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