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The construction of averaged semi-analytical planetary motion theory up to the third degree of planetary masses by means CAS Piranha

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The study of planetary systems orbital evolution is one of important problems of Celestial mechanics. In this work authors present the algorithm for the construction of the averaged semi-analytical motion theory up to the third degree of the small parameter for the case of planetary system with four planets. In this case the small parameter is the ratio of sum of planetary masses to the mass of the star.

The Hamiltonian of four-planetary problem is written in Jacobi coordinates and it is expressed into the Poisson series in elements of Poincare's second system. It allows sufficiently simplifying an angular part of the series expansion. In this case only one angular element – mean longitude, is defined.

The averaged Hamiltonian and the motion equations in averaging elements are constructed by Hori-Deprit method as the series in the small parameter and all orbital elements. The transformation between osculating and averaged orbital elements is performed by using of the functions for the change of variables. The using of the averaged motion equations allows sufficiently increase time step of the next numerical integration.

All analytical manipulations are performed by using of computer algebra system Piranha [1]. The author of Piranha system is Francesco Biscani from Max Plank Institute for Astronomy in Heidelberg, Germany. Piranha is echeloned Poisson series processor. It is developing C++ code with Python interface for analytical calculations with polynomials, Poisson series and echeloned Poisson series.

Orbital elements and masses are kept in the series expansions as symbol variables. It should be noted that series numerical coefficients are kept as rational numbers with arbitrary precision for the elimination of rounding errors.

The terms with the first order of the small parameter in the averaged Hamiltonian is constructed up to 8-th degree of eccentric and oblique Poincare elements. The second order terms is constructed up to 6-th degree and the third order terms – up to 2-nd degree of eccentric and oblique Poincare elements. It allows to get high precision motion equations for giant planets of Solar system and various extrasolar systems also. The algorithms of the expansion into the Poisson series and the construction of motion equations are presented in this work.

The results of numerical integration of the motion equations for the Sun – Jupiter – Saturn – Uranus – Neptune's system on a time interval of 10 billion years is considered. It is performed by Everhart method of 15-th order. The motion of the planets has an almost periodic character. The orbital eccentricities and inclinations save small values over whole time of the integration. The comparison with numerical theories is given.

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Keywords: CAS Piranha, echeloned Poisson series processor, four-planetary problem, semi-analytical motion theory, Hori-Deprit method, Jacobi coordinates, second system of Poincare elements

References

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